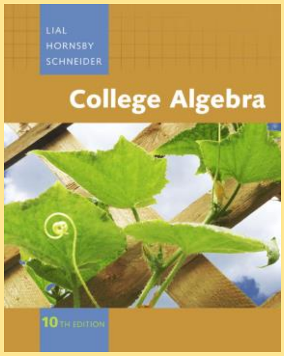
6

Analytic Geometry



6 Analytic Geometry

6.1 Parabolas

Conic Sections ▪ Horizontal Parabolas ▪ Geometric Definition and Equations of Parabolas ▪ An Application of Parabolas

6.1 Example 1 Graphing a Parabola with Horizontal Axis (page 607)

Graph $x – 4 = –(y + 2)^2$. Give the domain and range.

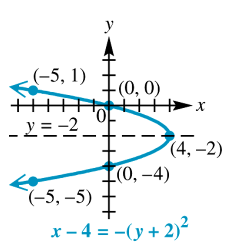
The graph has vertex (4, –2) and opens to the left because $a < 0$.

The graph is a reflection of the graph of $x = y^2$ across the y-axis with a translation 4 units to the right and 2 units down. Its axis is the line $y = –2$.

6.1 Example 1 Graphing a Parabola with Horizontal Axis (cont.)

Use the vertex and axis along with a few additional points to plot the graph.

x	y
–5	1
0	0
4	–2
0	–4
–5	–5



Domain: $(-\infty, 4]$ Range: $(-\infty, \infty)$

6.1 Example 2 Graphing a Parabola with Horizontal Axis (page 607)

Graph $x = \frac{1}{2}y^2 - 4y + 6$. Give the domain and range.

Complete the square on y to find the vertex and the axis.

$$x = \tfrac{1}{2}y^2 - 4y + 6$$
$$= \tfrac{1}{2}\left(y^2 - 8y + \quad\right) + 6 \qquad \text{Factor out } \tfrac{1}{2}.$$
$$= \tfrac{1}{2}\left(y^2 - 8y + 16 - 16\right) + 6 \qquad \text{Complete the square.}$$
$$= \tfrac{1}{2}\left(y^2 - 8y + 16\right) - \tfrac{1}{2}(16) + 6 \qquad \text{Distributive property}$$
$$= \tfrac{1}{2}(y-4)^2 - 2 \Rightarrow x + 2 = \tfrac{1}{2}(y-4)^2 \qquad \text{Factor; simplify.}$$

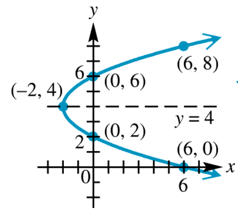
The graph has vertex (–2, 4) and opens to the right.

6.1 **Example 2** Graphing a Parabola with Horizontal Axis (cont.)

The graph is the graph of $x = \frac{1}{2}y^2$, translated 2 units to the right and 4 units up. Its axis is the line $y = 4$.

Use the vertex and axis along with a few additional points to plot the graph.

x	y
6	0
0	2
−2	4
0	6
6	8



Domain: $[-2, \infty)$

Range: $(-\infty, \infty)$

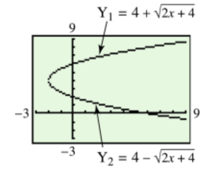
6.1 **Example 2** Graphing a Parabola with Horizontal Axis (cont.)

Graphing calculator solution

Write two equations by solving for y using the equation found in the algebraic solution.

$$x + 2 = \frac{1}{2}(y-4)^2 \Rightarrow 2x + 4 = (y-4)^2 \Rightarrow$$
$$\pm\sqrt{2x+4} = y - 4 \Rightarrow 4 \pm \sqrt{2x+4} = y$$

Graph the two equations. Their union is the graph of $x = \frac{1}{2}y^2 - 4y + 6$.



6.1 **Example 3(a)** Determining Information About Parabolas From Their Equations (page 609)

Find the focus, directrix, vertex, and axis of the parabola $x^2 = -4y$. Then use the information to graph the parabola.

$$x^2 = -4y \Rightarrow x^2 = 4py \Rightarrow 4p = -4 \Rightarrow p = -1$$

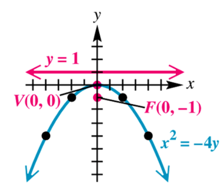
Since the x-term is squared, the parabola is vertical, with focus $(0, p) = (0, -1)$ and directrix $y = -p = -1$.

Vertex: (0, 0) Axis: $x = 0$

6.1 **Example 3(a)** Determining Information About Parabolas From Their Equations (cont.)

Use the vertex and axis along with a few additional points to plot the graph.

x	y
−4	−4
−2	−1
0	0
2	−1
4	−4



Domain: $(-\infty, \infty)$ Range: $(-\infty, 0]$

6.1 **Example 3(b)** Determining Information About Parabolas From Their Equations (page 610)

Find the focus, directrix, vertex, and axis of the parabola $y^2 = 12x$. Then use the information to graph the parabola.

$$y^2 = 12x \Rightarrow y^2 = 4px \Rightarrow 4p = 12 \Rightarrow p = 3$$

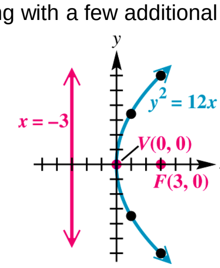
Since the y-term is squared, the parabola is horizontal, with focus $(p, 0) = (3, 0)$ and directrix $x = -p = -3$.

Vertex: (0, 0) Axis: $y = 0$

6.1 **Example 3(b)** Determining Information About Parabolas From Their Equations (cont.)

Use the vertex and axis along with a few additional points to plot the graph.

x	y
3	−6
1	$-2\sqrt{3} \approx -3.5$
0	0
1	$2\sqrt{3} \approx 3.5$
3	6



Domain: $[0, \infty)$ Range: $(-\infty, \infty)$

6.1 **Example 4(a)** Writing Equations of Parabolas (page 611)

Write an equation for a parabola with focus $\left(0, \frac{3}{4}\right)$ and vertex at the origin.

A parabola with focus $\left(0, \frac{3}{4}\right)$ and vertex at the origin is a vertical parabola with equation of the form $x^2 = 4py$.

$$p = \frac{3}{4} \Rightarrow \text{the parabola opens up.}$$

$$x^2 = 4\left(\frac{3}{4}\right)y \Rightarrow x^2 = 3y \text{ or } y = \frac{1}{3}x^2$$

6.1 **Example 4(b)** Writing Equations of Parabolas (page 611)

Write an equation for a parabola with a horizontal axis, vertex at the origin, and passing through the point (–4, –8).

A parabola with a horizontal axis, vertex at the origin, and passing through the point (–4, –8) is a horizontal parabola with equation of the form $y^2 = 4px$.

Substitute (–4, –8) into $y^2 = 4px$ to solve for p.

$$(-8)^2 = 4p(-4) \Rightarrow 64 = -16p \Rightarrow -4 = p$$

The equation of the parabola is $y^2 = -16x$.

6.1 **Example 5** Writing an Equation of a Parabola (page 611)

Write an equation for the parabola with vertex (–2, 3) and focus (–2, 2), then graph the parabola. Give the domain and range.

The focus is below the vertex, so the axis is vertical and the parabola opens downward.

The distance between the vertex and the focus is 3 − 2 = 1. Since the parabola opens downward, $p = -1$.

The equation is of the form $(x-h)^2 = 4p(y-k)$.

6.1 **Example 5** Writing an Equation of a Parabola (cont.)

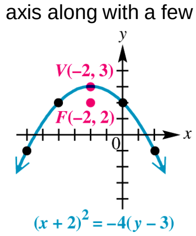
Substitute $p = -1$, $h = -2$ and $k = 3$ to find the equation.

$$(x-h)^2 = 4p(y-k)$$

$$(x-(-2))^2 = 4(-1)(y-3) \Rightarrow (x+2)^2 = -4(y-3)$$

Use the vertex and axis along with a few additional points to plot the graph.

x	y
−6	−1
−4	2
−2	3
0	2
4	−6

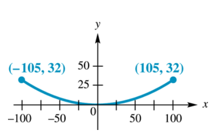


Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

6.1 **Example 6(a)** Modeling the Reflective Property of Parabolas (page 613)

The Parkes radio telescope has a parabolic dish shape with diameter 210 ft and depth 32 ft. The graph below models a cross section of the telescope. Using the graph, find the equation of the directrix of the parabola.



The equation of the parabola is $x^2 = 4py$.

6.1 **Example 6(a)** Modeling the Reflective Property of Parabolas (cont.)

Substitute $x = 105$ and $y = 32$, then solve for p.

$$x^2 = 4py$$

$$(105)^2 = 4p(32)$$

$$11{,}025 = 128p \Rightarrow p = \frac{11{,}025}{128} \approx 86.1$$

The equation of the directrix is $y = -86.1$.

6.1 Example 6(b) Modeling the Reflective Property of Parabolas (page 613)

What is the width of the parabolic dish 25 ft above the vertex?

From part (a), $p = \frac{11{,}025}{128}$, so the equation of the parabola is $x^2 = 4\left(\frac{11{,}025}{128}\right)y = \frac{11{,}025}{32}y.$

Let $y = 25$, then solve for x.

$$x^2 = \frac{11{,}025}{32}(25) \Rightarrow x = \pm 92.8$$

The width of the dish is 2(92.8) ≈ 185.6 ft.

6.2 Ellipses

Equations and Graphs of Ellipses ▪ Translated Ellipses ▪ Eccentricity ▪ Applications of Ellipses

6.2 Example 1(a) Graphing Ellipses Centered at the Origin (page 618)

Graph the ellipse $4x^2 + 25y^2 = 100$. Find the coordinates of the foci, and give the domain and range.

Write the equation in standard form: $\frac{x^2}{5^2} + \frac{y^2}{2^2} = 1$

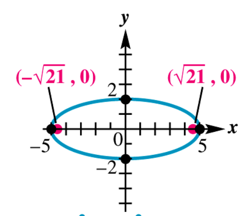
Center: (0, 0), $a = 5$, $b = 2$, so the intercepts are (±5, 0) and (0, ±2), and the major axis is horizontal.

Find the foci:

$$c^2 = a^2 - b^2 \Rightarrow c^2 = 25 - 4 \Rightarrow c^2 = 21 \Rightarrow c = \sqrt{21}$$

Foci: $\left(\pm\sqrt{21}, 0\right)$

6.2 Example 1(a) Graphing Ellipses Centered at the Origin (cont.)



$4x^2 + 25y^2 = 100$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$

Domain: [–5, 5] Range: [–2, 2]

6.2 Example 1(b) Graphing Ellipses Centered at the Origin (page 618)

Graph the ellipse $49x^2 + 9y^2 = 441$. Find the coordinates of the foci, and give the domain and range.

Write the equation in standard form: $\frac{x^2}{3^2} + \frac{y^2}{7^2} = 1$

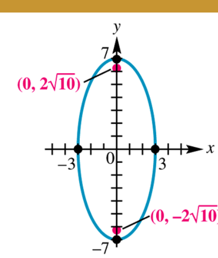
Center: (0, 0), $a = 3$, $b = 7$, so the intercepts are (±3, 0) and (0, ±7), and the major axis is vertical.

Find the foci:

$$c^2 = a^2 - b^2 \Rightarrow c^2 = 49 - 9 \Rightarrow c^2 = 40 \Rightarrow c = \sqrt{40} = 2\sqrt{10}$$

Foci: $\left(0, \pm 2\sqrt{10}\right)$

6.2 Example 1(b) Graphing Ellipses Centered at the Origin (cont.)



$49x^2 + 9y^2 = 441$

$\frac{x^2}{9} + \frac{y^2}{49} = 1$

Domain: [–3, 3] Range: [–7, 7]

6.2 Example 2 Writing the Equation of an Ellipse (page 619)

Write the equation of the ellipse having center at the origin, foci at (–5, 0) and (5, 0), and major axis with length 18 units.

Since the major axis has length 18 units, $2a = 18 \Rightarrow a = 9$.

$c = 5$, so $c^2 = a^2 - b^2 \Rightarrow 25 = 9^2 - b^2 \Rightarrow b^2 = 56.$

The equation of the ellipse is $\dfrac{x^2}{9^2} + \dfrac{y^2}{56} = 1 \Rightarrow \dfrac{x^2}{81} + \dfrac{y^2}{56} = 1.$

6.2 Example 3 Graphing a Half-Ellipse (page 620)

Graph $\dfrac{y}{3} = -\sqrt{1 - \dfrac{x^2}{4}}$. Give the domain and range.

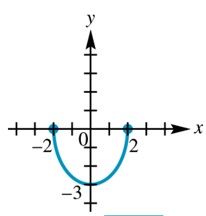
Square both sides of the equation:

$$\frac{y^2}{9} = 1 - \frac{x^2}{4} \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$$

This is the equation of a vertical ellipse with center (0, 0), x-intercepts ±2 and y-intercepts ±3.

Since $\dfrac{y}{3} = -\sqrt{1 - \dfrac{x^2}{4}}$, the only possible values of y are those making $\dfrac{y}{3} \le 0 \Rightarrow y \le 0$.

6.2 Example 3 Graphing a Half-Ellipse (cont.)



$\dfrac{y}{3} = -\sqrt{1 - \dfrac{x^2}{4}}$

Domain: [–2, 2] Range: [–3, 0]

6.2 Example 4 Graphing an Ellipse Translated Away From the Origin (page 621)

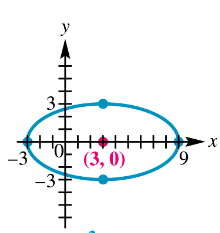
Graph $\dfrac{(x-3)^2}{36} + \dfrac{y^2}{9} = 1.$ Give the domain and range.

The center is (3, 0), $a = 6$, and $b = 3$.

The vertices are 6 units to the right and 6 units to the left of the center at (−3, 0) and (9, 0).

The endpoints of the minor axis are 3 units below and 3 units above the center at (3, 3) and (3, −3).

6.2 Example 4 Graphing an Ellipse Translated Away From the Origin (cont.)



$\dfrac{(x-3)^2}{36} + \dfrac{y^2}{9} = 1$

Domain: [–3, 9] Range: [–3, 3]

6.2 Example 5(a) Finding Eccentricity from Equations of Ellipses (page 622)

Find the eccentricity of the ellipse $\dfrac{x^2}{81} + \dfrac{y^2}{100} = 1.$

Since 100 > 81, $a^2 = 100$, which gives $a = 10$.

$$c = \sqrt{a^2 - b^2} = \sqrt{100 - 81} = \sqrt{19}$$

$$e = \frac{c}{a} = \frac{\sqrt{19}}{10} \approx .44$$

6.2 **Example 5(b)** Finding Eccentricity from Equations of Ellipses (page 622)

Find the eccentricity of the ellipse $\frac{x^2}{11}+\frac{y^2}{6}=1.$

Since 11 > 6, a^2 = 11, which gives $a=\sqrt{11}.$

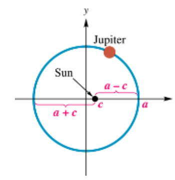
$$c=\sqrt{a^2-b^2}=\sqrt{11-6}=\sqrt{5}$$

$$e=\frac{c}{a}=\frac{\sqrt{5}}{11}\approx .67$$

6.2 **Example 6** Applying the Equation of an Ellipse to the Orbit of a Planet (page 622)

The orbit of Jupiter is an ellipse with the sun at one focus. The eccentricity of the ellipse is .0489, and the maximum distance of Jupiter from the sun is 507.4 million mi. Find the closest distance that Jupiter comes to the sun. (*Source: The World Almanac and Book of Facts*.)

The figure shows the orbit of Jupiter with the origin at the center of the ellipse and the sun at one focus. Jupiter is closest to the sun when Jupiter is at the right endpoint of the major axis and farthest from the sun when Jupiter is at the left endpoint.



6.2 **Example 6** Applying the Equation of an Ellipse to the Orbit of a Planet (cont.)

The maximum distance is *a* + *c* and the minimum distance is *a* − *c*.

$$a+c=507.4\Rightarrow c=507.4-a$$

$$e=\frac{c}{a}\Rightarrow\frac{507.4-a}{a}=.0489$$

$$507.4-a=.0489a$$

$$a\approx 483.75$$

$$c=507.4-483.75=23.65$$

so $a-c=483.7-23.65\approx 460.1$

The closest distance Jupiter comes to the sun is about 460.1 million miles.

6.2 **Example 7** Modeling the reflective Property of Ellipses (page 623)

If a lithotripter is based on the ellipse

$$\frac{x^2}{40}+\frac{y^2}{24}=1$$

determine how many units both the kidney stone and the source of the beam must be placed from the center of the ellipse.

The kidney stone and the source of the beam must be placed at the foci. a^2 = 40 and b^2 = 24 so $c=\sqrt{a^2-b^2}=\sqrt{40-24}=\sqrt{16}=4.$

The foci are at (–4, 0) and (4, 0).

The kidney stone and the source must be placed on a line 4 units from the center.

6.3 Hyperbolas

Equations and Graphs of Hyperbolas ▪ Translated Hyperbolas ▪ Eccentricity

6.3 **Example 1** Using Asymptotes to Graph a Hyperbola (page 628)

Graph $\frac{x^2}{4}-\frac{y^2}{25}=1$. Give the domain and range.

The equation is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, so it is centered at (0, 0) and has branches opening to the left and right.

The vertices of the hyperbola are (2, 0) and (–2, 0) since *a* = 2.

The equations of the asymptotes are $y=\pm\frac{5}{2}x.$

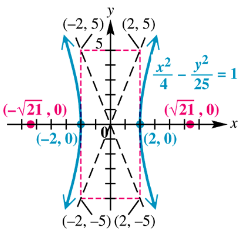
If *x* = 2, then *y* = ±5. If *x* = –2, then *y* = ±5. The corners of the fundamental rectangle are (2, 5), (–2, 5), (–2, –5), and (2, –5).

6.3 **Example 1** Using Asymptotes to Graph a Hyperbola (cont.)

Find the foci:

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 25 \Rightarrow c^2 = 29 \Rightarrow c = \sqrt{29}$$

The foci are $\left(-\sqrt{29}, 0\right)$ and $\left(\sqrt{29}, 0\right)$.



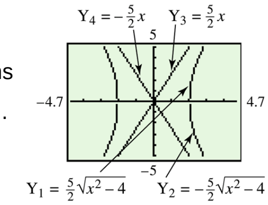
Domain: $(-\infty, -2] \cup [2, \infty)$
Range: $(-\infty, \infty)$

6.3 **Example 1** Using Asymptotes to Graph a Hyperbola (cont.)

Graphing calculator solution

Solve $\frac{x^2}{4} - \frac{y^2}{25} = 1$ for y: $y = \pm\frac{5}{2}\sqrt{x^2 - 4}$

The union of the two graphs is the graph of $\frac{x^2}{4} - \frac{y^2}{25} = 1$.



6.3 **Example 2** Graphing a Hyperbola (page 630)

Graph $16y^2 - 25x^2 = 400$. Give the domain and range.

Divide both sides of the equation by 400 to obtain

$$\frac{y^2}{25} - \frac{x^2}{16} = 1 \Rightarrow \frac{y^2}{5^2} - \frac{x^2}{4^2} = 1.$$

The equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, so it is centered at (0, 0) and has branches opening upward and downward.

The vertices of the hyperbola are (0, 5) and (0, –5) since $a = 5$.

The equations of the asymptotes are $y = \pm\frac{5}{4}x$.

6.3 **Example 2** Graphing a Hyperbola (cont.)

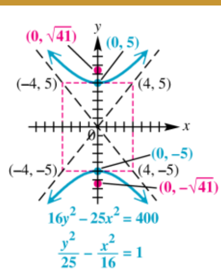
The corners of the fundamental rectangle are (–4, 5), (4, 5), (4, –5), and (–4, –5).

Find the foci:

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 25 + 16 \Rightarrow c^2 = 41 \Rightarrow c = \sqrt{41}$$

The foci are $\left(0, -\sqrt{41}\right)$ and $\left(0, \sqrt{41}\right)$.

6.3 **Example 2** Graphing a Hyperbola (cont.)



Domain: $(-\infty, \infty)$ Range: $(-\infty, -5] \cup [5, \infty)$

6.3 **Example 3** Graphing a Hyperbola Translated Away From the Origin (page 631)

Graph $\frac{(x-2)^2}{16} - (y+4)^2 = 1$. Give the domain and range.

The equation is of the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ where $h = 2$, $k = -4$, $a = 4$, and $b = 1$. The graph opens to the right and left.

The vertices are 4 units left and right of the center (2, –4) at (6, –4) and (–2, –4).

The equations of the asymptotes are

$$y = k \pm \frac{b}{a}(x - h) \Rightarrow y = -4 \pm \frac{1}{4}(x - 2)$$

6.3 Example 3 Graphing a Hyperbola Translated Away From the Origin (cont.)

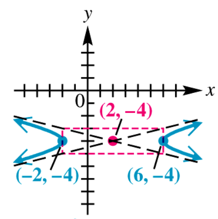
The corners of the fundamental rectangle are (6, –3), (6, –5), (–2, –3), and (–2, –5).

Find the foci:

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 16 + 1 \Rightarrow c = \sqrt{17}$$

The foci are $\sqrt{17}$ units left and right of the center at $\left(2-\sqrt{17}, -4\right)$ and $\left(2+\sqrt{17}, -4\right)$.

6.3 Example 3 Graphing a Hyperbola Translated Away From the Origin (cont.)



$$\frac{(x-2)^2}{16} - (y+4)^2 = 1$$

Domain: $(-\infty, -2] \cup [6, \infty)$ Range: $(-\infty, \infty)$

6.3 Example 4 Finding Eccentricity from the Equation of a Hyperbola (page 632)

Find the eccentricity of the hyperbola $\frac{y^2}{100} - \frac{x^2}{81} = 1$.

$a^2 = 100$, so $a = 10$.

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 100 + 81 \Rightarrow c = \sqrt{181}$$

$$e = \frac{c}{a} = \frac{\sqrt{181}}{10} \approx 1.3$$

6.3 Example 5 Finding the Equation of a Hyperbola (page 632)

Find the equation of the hyperbola with eccentricity 3 and foci at (–2, 5) and (–2, –3).

The foci have the same *x*-coordinate, so the hyperbola is vertical.

The center of the hyperbola is halfway between the foci, at (–2, 1).

The distance from each focus to the center is 4, so $c = 4$.

6.3 Example 5 Finding the Equation of a Hyperbola (cont.)

Use the eccentricity to find *a*:

$$e = \frac{c}{a} \Rightarrow 3 = \frac{4}{a} \Rightarrow 3a = 4 \Rightarrow a = \frac{4}{3}$$

Now find the value of b^2 given $c^2 = a^2 + b^2, a = \frac{4}{3}$, and $c = 4$.

$$c^2 = a^2 + b^2 \Rightarrow 4^2 = \left(\frac{4}{3}\right)^2 + b^2 \Rightarrow 16 = \frac{16}{9} + b^2$$

$$\Rightarrow b^2 = 16 - \frac{16}{9} = \frac{128}{9}$$

6.3 Example 5 Finding the Equation of a Hyperbola (cont.)

$$a = \frac{4}{3} \Rightarrow a^2 = \frac{16}{9}, b^2 = \frac{128}{9}, h = -2, \text{ and } k = 1$$

The equation of the hyperbola is $\frac{(y-1)^2}{\frac{16}{9}} - \frac{(x+2)^2}{\frac{128}{9}} = 1$

or $\frac{9(y-1)^2}{16} - \frac{9(x+2)^2}{128} = 1.$

6.4 Summary of the Conic Sections

Characteristics ▪ Identifying Conic Sections ▪ Geometric Definition of Conic Sections

6.4 Example 1(a) Determining Types of Conic Sections from Equations (page 639)

Determine the type of conic section represented by the equation and graph it.

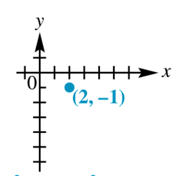
$$x^2 - 4x + y^2 + 2y = -5$$

Complete the squares on x and y:

$$(x^2 - 4x + 4) + (y^2 + 2y + 1) = -5 + 4 + 1$$
$$(x-2)^2 + (y+1)^2 = 0$$

This is the equation of a circle centered at (2, –1) with radius 0, that is, the point (2, –1).

6.4 Example 1(a) Determining Types of Conic Sections from Equations (cont.)



$x^2 - 4x + y^2 + 2y = -5$

6.4 Example 1(b) Determining Types of Conic Sections from Equations (page 639)

Determine the type of conic section represented by the equation and graph it.

$$9x^2 = 144 - 16y^2$$

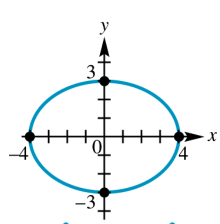
Write the equation in standard form:

$$9x^2 = 144 - 16y^2 \Rightarrow 9x^2 + 16y^2 = 144 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$$

The equation is in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ with $a = 4$, $b = 3$, $h = 0$, and $k = 0$.

This is the equation of an ellipse centered at (0, 0) with x-intercepts (–4, 0) and (4, 0) and y-intercepts (0, –3) and (0, 3).

6.4 Example 1(b) Determining Types of Conic Sections from Equations (cont.)



$9x^2 = 144 - 16y^2$

6.4 Example 1(c) Determining Types of Conic Sections from Equations (page 639)

Determine the type of conic section represented by the equation and graph it.

$$2x + y^2 - 10y + 17 = 0$$

Since only one variable is squared, the equation represents a parabola. Get the term with x (the variable that is not squared) alone on one side.

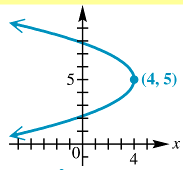
$$y^2 - 10y + 17 = -2x$$
$$(y^2 - 10y + 25) + 17 = -2x + 25 \quad \text{Complete the square.}$$
$$(y-5)^2 = -2x + 8$$
$$(y-5)^2 = -2(x-4)$$

6.4 **Example 1(c)** Determining Types of Conic Sections from Equations (cont.)

The equation is of the form $x-h=a(y-k)^2$ with $a=-\frac{1}{2}, h=4,$ and $k=5.$ It is a parabola with vertex (4, 5) and horizontal axis y = 5. It opens to the left.

Use the vertex and axis and a few additional points to plot the graph.

x	y
–4	1
2	3
4	5
2	7
–4	9



$2x + y^2 - 10y + 17 = 0$

 6-55

6.4 **Example 1(d)** Determining Types of Conic Sections from Equations (page 639)

Determine the type of conic section represented by the equation and graph it.

$$-4x^2-24x+25y^2+100y=36$$

Complete the squares on *x* and *y*.

$$-4x^2-24x+25y^2+100y=36$$

$$-4\left(x^2+6x+9\right)+25\left(y^2+4y+4\right)=36-36+100$$

$$-4(x+3)^2+25(y+2)^2=100$$

$$\frac{(y+2)^2}{4}-\frac{(x+3)^2}{25}=1$$

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6.4 **Example 1(d)** Determining Types of Conic Sections from Equations (cont.)

The equation is of the form $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ with *a* = 2, *b* = 5, *h* = –3, and *k* = –2.

$$c^2=a^2+b^2 \Rightarrow c^2=2^2+5^2 \Rightarrow c=\sqrt{29}$$

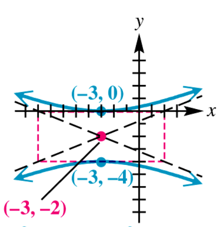
The equations of the asymptotes are

$$y=k\pm\frac{b}{a}(x-h) \Rightarrow y=-2\pm\frac{2}{5}(x+3)$$

This is the equation of a hyperbola centered at (–3, –2), vertices (–3, 0) and (–3, –4), asymptotes $y=-2\pm\frac{2}{5}(x+3)$ and foci $\left(-3,-2+\sqrt{29}\right)$ and $\left(-3,-2-\sqrt{29}\right)$.

 6-57

6.4 **Example 1(d)** Determining Types of Conic Sections from Equations (cont.)



$-4x^2 - 24x + 25y^2 + 100y = 36$

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6.4 **Example 2** Determining Types of Conic Sections from Equations (page 639)

Identify the graph of $9x^2-72x+25y^2-100y=-19$.

$$9x^2-72x+25y^2-100y=-19$$

$$9\left(x^2-8x+16\right)+25\left(y^2-4y+4\right)=-19+9(16)+25(4)$$

Factor, then complete the squares on *x* and *y*.

$$9(x-4)^2+25(y-2)^2=225$$

$$\frac{(x-4)^2}{25}+\frac{(y-2)^2}{9}=1$$

This is the equation of an ellipse centered at (4, 2) with major axis endpoints (–1, 2) and (9, 2) and minor axis endpoints (4, –1) and (4, 5).

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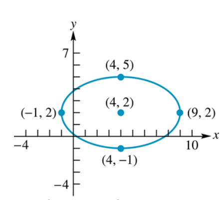
6.4 **Example 2** Determining Types of Conic Sections from Equations (cont.)



$9x^2 - 72x + 25y^2 - 100y = -19$

$\frac{(x-4)^2}{25}+\frac{(y-2)^2}{9}=1$

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